The International Conference of Computational Methods
April 4-6, 2007, Hiroshima

STRESS-BASED CROSSOVER OPERATOR FOR STRUCTURAL TOPOLOGY OPTIMIZATION

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Abstract

Evolutionary structural optimization (ESO) is based on the concept of slowly removing inefficient materials from a structure so that the residual structure evolves toward the optimum. As ESO is not an optimization algorithm that constructs structures using an objective function, the design obtained does not necessarily to satisfy the requirements of the designers. In addition, when there are constraints (such as displacement and stress) it is difficult for ESO to guarantee the constraints during iteration. Genetic Algorithms (GA) represents a powerful global stochastic search method that has been applied to a variety of engineering design problems. GA can also be used to solve problems with various constraints. Based on observations with ESO we propose a stress-based crossover operator for structural topology optimization problems on constrained displacement and constrained stress. Experiments indicate that this crossover operator can significantly reduce the "checkerboard" pattern that often arises on application if GA to structural topology optimization. For constrained problems, various violation penalty functions are often adopted to drive the search direct toward the optimal topology. In this study as we dragged back the violated individuals gradually we attempted to maintain the diversity of the population, which is important for GA. Application to the 2D-cantilever problem showed that the proposed crossover can search various applicable and beautiful topologies more quickly than two-point crossover GA.

Keywords: GA, ESO, Stress-based Crossover, Structural Topology Optimization

Introduction

Structural topology optimization involves searching for an optimal material layout in engineering. Formal methods addressing this problem include the homogenization method [1] in which each element in a grid contains composite material of continuously variable density [0,1] and orientation. Xie and Steven [2] proposed the evolutionary structural optimization (ESO) method that follows the concept of gradual removal of inefficient material from a structure. As an extension of the ESO method, bi-directional ESO (BESO) allows efficient materials to be added in addition to removal of inefficient materials to remedy the elements deleted in previous processes [3]. For structural topology optimization, homogenous and density methods suffer from the problem that a solution may converge on a local minimum. Evaluation optimization algorithms, such as genetic algorithms, simulated annealing (SA), evolution strategy (ES), and the tabu search algorithm, have been widely used for various optimization problems [4]. Evaluation approaches to continuum topology optimization problems have been developed in recent decades. Although ESO is not a search algorithm and cannot be applied to constrained problems, it can obtain a beautiful connected topology that is important for practical applications. GA is a powerful global stochastic search method and has been applied to a variety of engineering design problems. GA can also be used for problems with various constraints. The binary genotype and the stochastic search often produce the "checkerboard" problem, which makes the results difficult to realize. Based on observations with ESO, we propose a stress-based crossover operator (SX) for structural topology optimization problems in constrained displacement and constrained stress.

The effectiveness of this approach was verified by application to a 2-dimensional cantilever problem. Comparison with two-point crossover GA (SGA) showed that SX has following advantages: 1) acceleration of the convergence speed. 2) topology with less checkerboard problem. 3) searching out various practical topologies.

This paper starts with a description of the optimization problem formulation. This is followed by a brief review of ESO. Next, we present an introduction of GA applied to structural topology where our proposed stress-based crossover is described in detail. Finally, the optimization results of SX and SGA and ESO are presented and discussed.

Problem Description and Fitness Assignment

The 2-dimensional cantilever problem shown in Figure 1 was used to study the effectiveness of evolutionary algorithms proposed in this paper. For problem, the left edge is clamped with a concentrated loading (10GN) applied at the mid-point of the right side. The problem can be described as shown in formula (1). The objective function is to minimize the weight of the plate with the constraints, stress and displacement. The values of the constraints used in this paper are shown in Table 1. $Stress_{\lim}$ and $Disp_{\lim}$ are the constrained equivalent stress and constrained displacement, respectively.

$$\min . f(x) = \sum_{i=0}^{N} x_i \quad , \quad x_i \in \{0,1\} \qquad (1)$$
$$\text{subject to}: Stress_{\max} < Stress_{\lim}, Disp_{\max} < Disp_{\lim}$$

Table 1 Constraint Conditions

Constraints	Value
$Stress_{\lim}$	5.5e10
$Disp_{\lim}$	10.0

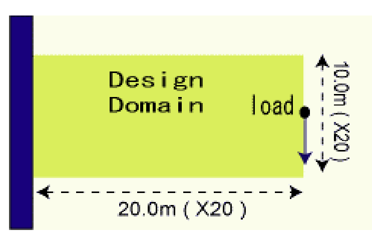

Figure 1. 2D-Cantilever Plate

ESO

ESO has been effectively addressed by Xie and Steven[1993]. There are two important parameters in ESO: rejection rate (rrRate) and evaluation rate (erRate). Initially, the rrRate and erRate are assigned small values. For one individual, the elements of which stress fits the formula (2) will be deleted in every generation. Here, $p_i.Stress[k]$ is the stress of element-k of individual p_i. If there is no element deleted at the current rrRate, the rrRate is adjusted according to formula (3):

$$p_i.Stress[k] / Stress_{\max} < rrRate; i = 0...n \tag{2}$$

$$rrRate = rrRate + erRate \tag{3}$$

There are no good methods to decide which value is best for ESO, except the performance of many experiments. However most researchers adopt small values. In our study, the values used were 0.005 and 0.001 for the initial rrRate and erRate, respectively. The topology with a weight of 140, which is named ESO, is shown in Figure 3..

GA Operators and Procedures

Genetic algorithms are highly effective search algorithms based on the principles of genetics and natural selection [4]. GA represents a class of global search algorithms. There are three significant operators for GA-selection, crossover and mutation. Crossover is the predominant operator in GA, which drives the global search direction. Many different crossover operators have been devised to date. A mutation operator is seen as a “background” operator responsible for re-introducing mistakenly lost gene values, preventing genetic drift, and providing a small element of random convergence [5]. However for structural topology optimization, binary encoded mechanisms often cause the “checkerboard” problem. Based on observations with ESO algorithm, we propose a stress-based crossover operator, the aim of which is to combine the global search ability of GA and applicable geometry of the results of ESO.

In this section, firstly we introduce the encoded method for structural optimization and then describe our stress-based crossover (SX) operator in detail.

Chromosome Representation

The straightforward and natural method is the bit-string or bit-array representation. Kane & Schoenauer discussed these two representations as well as the operators [6]. Recently, more advanced forms of representations for continuum topology optimization design problems have been proposed, including Voronoi-based representations [7], which are based on the concepts of Voronoi diagrams studied in computational geometry. In addition, Hamda[8] considered a continuum TOD as an evolutionary multi-objective optimization problem. Kim and Week introduce a variable chromosome length genetic algorithm in topology optimization [9].

In this study, the bit-string representation was adopted as the population chromosome representation to define the distribution of material and voids in a two-dimensional topology design domain, in which '1' represents material and '0' void.

Crossover Operator

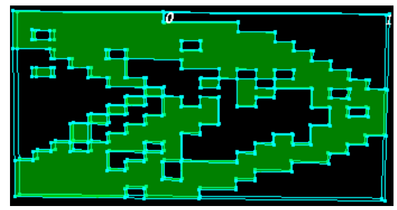

Figure 2. Checkerboard Problem

Binary encoded GA for structural topology optimization often result in the formation of non-analyzable (disconnected) structures and checkerboard patterns, as shown in Figure 2. Additional strategies must be used to bias the formation of connected structures during the GA iterations to improve performance. Therefore, based on observations with ESO, we propose a stress-based crossover operator called SX from here. The procedures of this operator are as follows. First, the nomenclatures used in this operator are explained.

$P(t)=\{p_i(t) \mid i\in\{1\ ...n\}\}$ is the population of generation t, n is the population size.

$p_i(t).code[k]$ is one gene of chromosome p_i(t), where $k\in\{1...N\}$, N is chromosome length.

$p_i(t).stress[k]$ is the stress of gene k .

$p_i(t+1).power[k]$ is the power of the gene k of child individual $p_i(t+1)$.

1. For each individual $p_i(t),(i=1...n)$, randomly select one individual $p_j(t),(j\neq i)$.
2. Calculate the power of every gene of the child individual using formula (4):

$$p_i(t+1).power[k]=p_i(t).stress[k]+p_j(t).stress[k]\ \ ,k=1\ldots N \quad (4)$$

3. Sort the genes of the child individual according to $p^{'}.power[k]$. Then, divide the genes into two groups, U1 and U0, according to the power of every gene according to formula (5). The size of U1 is equal to the material number of p_i. The size of U0 is equal to the (N-weight).

$$p_i(t+1).code[k]=\begin{cases}1 & ,\ if\ p_i(t+1).power[k]\ \in U1.\\ 0 & ,if\ p_i(t+1).power[k]\ \in U0\end{cases} \quad (5)$$

Fitness Function and Constraint Handling Strategy

After a chromosome is mapped into the design domain and finite element analysis is performed, the fitness of each individual will be calculated. For problems with constraints, during the evaluation some individuals may be outside the design domain. In our study, the fitness function is defined according to formula (6), where the smaller of the fitness values the better of this individual is. In the fitness function s1 and s2 are the coefficients. $Stress_{max}$ is the maximal stress of the current individual. $Disp_{max}$ is the displacement of the current individual. In this paper, the displacement of the loading point in the loading direction is substituted for the maximal displacement of current individual. Generally for reasonable individuals the ratio of $Stress_{max}/Stress_{lim}$ and $Disp_{max}/Disp_{lim}$ are less than 1 so the fitness function focuses on the weight. Once the individual violates the constraints, the proportion of the effect of stress or displacement in the fitness function will increase. In other word, once the individuals are outside the design domain the violated individuals are assigned a bigger fitness values.

$$fitness(x) = weight + 10^{s1}\frac{Stress_{\max}}{Stress_{\lim}} + 10^{s2}\frac{Disp_{\max}}{Disp_{\lim}}$$
$$s1, s2 = \begin{cases} 1 & , \textit{if the constrains violated} \\ 0 & , \textit{others} \end{cases} \quad (6)$$

However For continuous structural topology optimization problem, it is common that the individuals may be outside the design domain in GA. Some researchers wish to find mechanisms to drag back individuals that are outside the design domain. For GA, once the diversity of population decreases the evaluation will slow down. In this study, as we applied a mechanism on violated individuals to draw them back into the design domain, we attempted to maintain the diversity of the population. We used the logical "OR" operator on the violated individuals. Along with increasing of the material in individual, the individual is dragged back gradually.

Results Comparison and Discussion

This section summarizes and discusses the comparative results obtained under various conditions. Initially, all the tests are performed on the 20×20 regular mesh with the same material properties. For all examples, the GA parameters are shown in Table 2. A number of experiments were performed to verify the efficiency of our proposed operator.

Table 2. Parameters of GA

Population Size	Chromosome Length	Elites	Crossover Rate	Mutation Rate	Tournament Size	Max. Generation
100	400	1	1	0.01	2	500

Geometry Comparison

To investigate the efficiency of our proposed crossover operator, we provide a comparison of SX with SGA and ESO with regard to geometry topology and the evaluation procedure.

The last topologies of SGA and SX are shown in Figure 3. Among the topologies, SGA was the best topology of simple GA with a two-point crossover; ESO is the result of ESO with weight 140; The results of SX are as shown from SX-a to SX-h. The properties of these topologies are shown in Table 3. Comparison of the geometry topologies indicated that in 500 generation SX searches out much more beautiful geometry topologies than SGA. On the other hand, the weights of the topologies of SX are lighter than that of SGA. From the comparison of ESO with SX-a and SX-b with regard to the similarity of

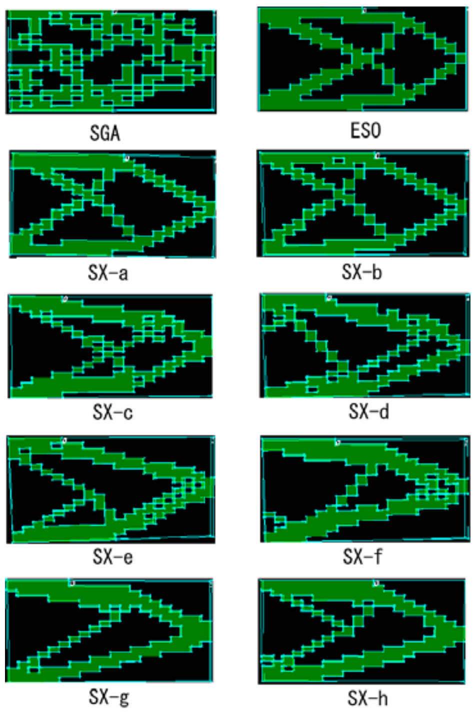

Figure 3. The Best topologies

geometry topologies, weight, stress and displacement of SX are much smaller than those of ESO. Among all the results for SX, SX-b is the most efficient topology.

Table 3. Properties of Topologies

Index	Weight	StressMax.	Displacement
SGA	176	4.9849e+10	9.8943
ESO	140	5.2773e+10	31.964
SX-a	134	3.7139e+10	9.0793
SX-b	124	4.3031e+10	9.9788
SX-c	138	4.4914e+10	9.8073
SX-d	135	4.2462e+10	9.9228
SX-e	130	4.1569e+10	9.9869
SX-f	140	5.1004e+10	9.9978
SX-g	132	4.5491e+10	9.9830
SX-h	142	4.1573e+10	9.5997

Evaluation Procedure Comparison

To further study the efficiency of SX, we compared SX with SGA with regard to weight and fitness shown in Figure 4. The evaluation speeds of SX on weight and fitness were faster than those of SGA.

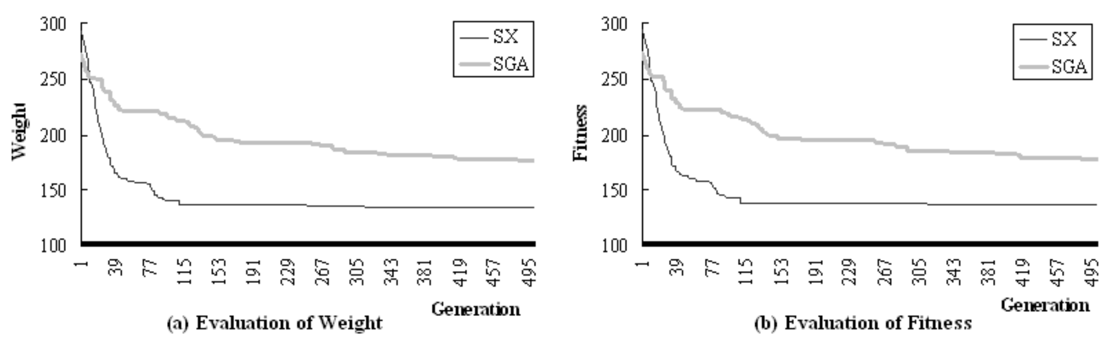

Figure 4. Evaluation Procedure

Conclusions and Future Work

A high performance stress-based crossover operator-SX of GA for structural topology optimization was proposed to combine the merits of ESO with regard to connected geometry and those of GA with regard to global search ability. Experiments and comparisons of SX with SGA on a 2-D cantilever problem showed that our proposed crossover operator is more efficient than the two-point crossover with regard to numerical results, geometry topologies, and evaluation curves.

Many points regarding SX require further study. First to increase speed and obtain a much wider search space, it is necessary to determine how best to decide the material number of child individuals. Second, athough SX can obtain more beautiful geometry topology than SGA, the “void” problem like in Figure 3 SX-b remains. Third, for constrained problems, during evoluation of GA some of the individuals may be outside the design domian, and further discussion of how best to deal with this problem is required. Finally, for GA, which individual will pass to the next generation is based on the fitness value only. Especially for structural topology optimization, binary-encoded GA can obtain some individuals of same weight, same stress and same displacement but different geometry topologies. Under these conditions, it is difficult to decide which is the best by the fitness function shown in this paper. Thus further research is required to determian how to define a fintess function that can include all the necessary and important information.

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