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DIVIDED RANGE GENETIC ALGORITHMS IN MULTIOBJECTIVE OPTIMIZATION PROBLEMS

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Abstract

In this paper, Divided Range Genetic Algorithm in Multi objective optimization Problems (DRGA) is proposed. In this method, population of GAs is sorted with respect to the objective function and divided into sub populations. In this model, the Pareto optimum solutions which are close to each other are collected by one sub population. Therefore, by this algorithm, the calculation efficiency is increased, and the neighborhood search can be performed. Through the numerical examples, the followings are made cleared . DRGA is very suitable GA model for parallel processing. DRGA can derive the good solutions compared to the single population model and the distributed model.
Keywords: Multi Objective Problems, Genetic Algorithms, Distributed Processing, Parallel Processing

1. INTRODUCTION

Genetic Algorithm (GA) is one of the random search methods and simulates the mechanism of heredity and evolution of creatures (Goldberg,1989). The usual optimization methods are the kinds of gradient methods. Therefore, it is difficult to find the global optimum, when there are several peaks in objective functions or when the objective function is not continuous. On the other hand, the GA can be applied to the problems where the objective function is discrete and there are several peaks.

There are several studies that concerned with the GA applied to the multi objective function (Fonseca and Fleming, 1995; Tamaki, et al., 1996; Coelo, 1999). Because the GA is one of the multi searching methods, it is suitable for finding the Pareto optimum solutions. There are several models are proposed for the multi objective GA. Schaffer developed the VEGA (Schaffer, 1985) . Goldberg et al. introduced the ranking method (Goldberg,1989) and Fonseca et al. also developed the MOGA (Fonseca and Fleming, 1993). In their methods, the Pareto optimum solutions are treated explicitly. Tamaki et al. (Tamaki, et al., 1996) introduced their model where the VEGA used and the Pareto optimum individuals are remained. [1] Additionary, there is a method of Murata et al. (Murata, et al., 1995). In their method, by weighting the values to each objective functions, they converted the objective optimization problems to single objective optimization problems.

Like this way, there are several models of multi objective GA and they can derive the good Pareto optimum solutions. However, it needs a lot of iterations to calculate the values of objective functions and the constrains. This leads to the high calculation costs. One of the solutions of this problem is to perform the multi objective GA in parallel processing.

There are several studies that concerned with the prallelization methods of GA for single object (Nang and Matsuo, 1994; Cantu-Paz, 1999, Sawai and Adachi, 1999). On the other hand, there are few studies of GA for multi objective optimization problems. The models of those studies are the same as that of GA for single objective. There is a model where the parts of evaluation of fitness are performed in parallel (Jones and Crossley, 1998). There is another model where the total population is divided into sub populations and the multi objective optimization is performed in each sub population (Vicini, 1998). However, the mechanism of searching the optimum is different between the single objective GA and the multi objective GA. In the single objective GA, only one optimum should be derived. Therefore, the diversity of the searching point is important in the first stage and the local search is important in the latter stage. On the other hand, in the multi objective GA, the both the diversity and the local search are important for all stages, because it should derive not one point but the meeting of the points. This fact suggests that the model which is different from the model used in the single objective GA should be used for the multi objective optimization in parallel.

[1] Tamaki et al. called the individuals that are in the Pareto front as the Pareto optimum individuals.

In this study, the new model of multi objective genetic algorithm for parallel processing. This model is called Divided Range Genetic Algorithm: DRGA. In the DRGA, the individuals are divided into sub populations by the values of their objective function. Therefore, the efficient search can be performed and the adequate local search also carried out. The introduced DRGA is applied to the numerical test problems and the validity of the model and the characteristics of the solutions are discussed.

2. MULTI OBJECTIVE GENETIC ALGORITHM

2.1 Multi Objective Optimization Problems

In the optimization problems, when there are several objective functions, the problems are called the Multi-objective Optimization Problems: MOPs.

The multi objective optimization problems are formulated as follows. In general,

$$min[f_1(x), f_2(x), \ldots, f_n(x)] \qquad (1)$$

$$subjecttog_i(x) \leq 0 \qquad (1, 2, \ldots, m) \qquad (2)$$

where $x \in F$ is the design variables and F is the domain that satisfies the constraints and is called the feasible domain.

Usually, there are trade off relations between the objective functions. Therefore the optimum solution is not only one. In this case, the concept of the Pareto optimum solution is introduced in the multi objective optimization problems (Ben-Tai, 1980).

(1) **Pareto dominant:**
When $x^1 \in F$ and $x^2 \in F$ satisfy $f_i(x^1) \leq f_i(x^2)$ for all of the objective functions f_i and satisfy $f_i(x^1) \in f_i(x^2)$ for some of the objective functions f_i, x^1 is dominant to x^2.

(2) **Pareto optimum solutions:**
When $x^1 \in F$ does not exist that dominant to x^0, x^0 is the Pareto optimum solutions.

In the real world problems, the multi objective optimization problems are often found, such as the design problems. In these problems, the objective optimizations have the trade off relation ships. Usually, these relation is not clear. Thus, when the relation can be grasped, the problem turns easily for the designers. Then, the deriving the Pareto optimum solutions is one of the goals in the multi objective optimization problems.

2.2 Parallelization of Multi Objective Genetic Algorithms

The genetic algorithm (GA) is an optimization method that mimics the process of evolution. GA is one of the multi point search methods. These points are called individuals and the assembly of the individuals are called population. The number of individuals are usually called population size. The new searching point is generated by the genetic operations of the crossover or the mutation. Each individual has the fitness value and is determined to survive corresponding to the fitness value. GA can find the solution by iterations of this cycle. This cycle is called the generation.

As it is explained, the methods of the multi objective genetic algorithms are classified into two categories: the method where the Pareto optimum individuals are treated explicitly and implicitly. In the method where the Pareto optimum individuals are treated explicitly, the individuals that are close to the Pareto solutions have high possibility to survive. The possibility is determined with the value of fitness function. Usually ranking method (Goldberg, 1989; Fonseca and Fleming, 1993) is used to determine the fitness value. The individuals that are dominant to the other individuals are rank 1. The individuals except for rank 1 individuals are rank 2, and so on.

The models of the multi objective genetic algorithms are classified into two categories: the model where the each genetic operation is performed in parallel and the model where the population is divided into the sub populations and the GA is performed in each sub population.

Among the models where the each genetic operation is performed in parallel, the most efficient way is the model where the evaluation operation is performed in parallel. This model is very useful, because, In the GA, it takes much time in evaluating the fitness function(Jones and Crossley, 1998). However, this model need a lot of message passings and this leads to the high network costs. At the same time, because this is one of the master and slave models, one CPU should be occupied as the master. Then the parallel efficiency decreases.

The model where the population is divided into sub populations is often called the distributed genetic algorithm (DGA) or the island model. The sub population is called an island and the GA is performed in each island. After the certain generations (this is called the migration interval), some individuals are chosen randomly and moved to the other islands. This operation is called the migration. The number of the individuals that migrate to the other island is determined by the multiplication the population size and the migration rate. There are several types of the island models. For example, Vicini (Vicini, 1998) introduced his island model where the population size of each island is different and the operations are performed in parallel.

In the single objective problems, the GA should find the only one global optimum. In the DGA, it has the two types of mechanism. One of them is the quick convergence. Because the population size is reduced in each island, the convergence is done quickly. The other mechanism is the maintenance of the diversity. Even each solution in each island is local, there are several solutions exist in islands, the diversity is not lost. These two mechanism help well to find the one global solution.

On the other hand, in the multi objective problems, the GA should find the assembly of the points. Therefore, the mechanism of the DGA does not act efficiently. The DGA can not perform the adequate local search in each island because the population size is small. At the same time, every island is searching in the same feasible domain, this leads to the waste of calculations.

The DRGA that is introduced in this study is one of the distributed population model. Therefore, the total population is divided into sub populations. However, the DRGA is overcome the problems of the DGA. The population is divided corresponding to the searching area. Therefore, the DRGA can adequate local search without waste calculations. the DRGA is explained precisely in the next chapter.

2.3 Matrix

One of the most difficult problems in multi objective optimization problems is to evaluate the Pareto optimum individuals. Because the Pareto optimum individuals are the assembly of the points, there is no good quantitative way of evaluating. Many researchers only show the derived Pareto optimum individuals in figures. This evaluation is not quantitative way and can only apply for two and tree objective functions.

Hiyane (Hiyane, 1997) introduced his matrix for the accuracy and the quality of the Pareto optimum individuals. In this study, the matrix that is simplified the Hiyane's methods are utilized as follows.

2.3.1 Error

When the real Pareto optimum solutions are given, the average of Euclid distance between the real Pareto solutions and each Pareto optimum individuals. When the error is small, the Pareto optimum individuals are very close to the real Pareto solutions. This matrix only can apply to the problem where the Pareto solutions are given. In this study, we used the shorthand expression of errors. In the test functions that are used in this study, the Pareto solutions exist on the constraints. Therefore, when $g(x) = 0$ is the real Pareto solutions, the following shorthand errors are used.

$$Error = \sqrt{\sum_{i=1}^{N} g(x_i)^2/N} \tag{3}$$

In this expression, N expresses the number of the Pareto optimum individuals.

2.3.2 Cover rate

Because the Pareto solutions are the assembly of the points, it sometimes happen that the error is 0.0 but the solutions are concentrated on one point. Therefore, the index of the diversity of the solutions is necessary. The cover rate is the index for this purpose. In Fig. 1, the concept of the cover rate are shown, when there are two objective functions. To derive the cover rate, the following procedures are taken. At first, the maximum and minimum values of one objective function is searched. Secondly, the distance between the maximum and the minimum is divided into the certain number of the division. Thirdly, the division area that have the Pareto optimum individuals is counted. Fourthly, the counted number is divided by the number of division. When every divided area has at least one Pareto optimum individual, this number becomes 1. When there are no area that has the Pareto optimum individuals, this number becomes 0. Fifthly, these steps are treated for every objective function. Finally, the cover rate is determined to average the number of each objective function. When the cover rate is close to 1, it means that the Pareto optimum individuals are not concentrated on one point and they spreads. It can be said that the Pareto solutions can be shown 50 points for one objective function (this means that there are 50*50 points are need when there are three objective functions), the number of division is 50 in this study.

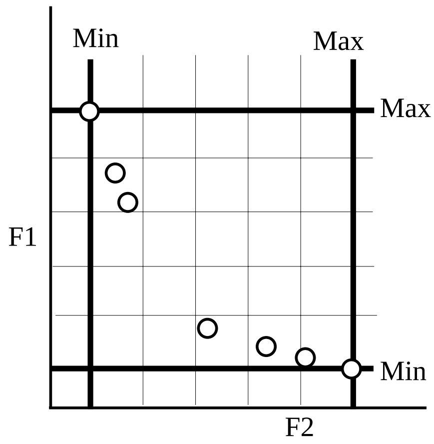

Fig. 1 Cover rate

2.3.3 Calculation time and number of objective function calls

When the accuracy is the index of the terminal condition, the same accuracy of solutions is derived. In this case, the calculation time or the number of objective function calls are important matrix. When the method can derive the same accuracy of the solutions with short calculation time and the small number of objective function calls, this method can be said the effective method.

3. DISTRIBUTED GENETIC ALGORITHMS

3.1 Overview of DRGA

In this study, the new model of the multi objective genetic algorithm is proposed. That is Divided Range Genetic Algorithm: DRGA. This model is suitable for parallel processings.

The multi objective genetic algorithm should have the following abilities to derive the good Pareto optimum solutions efficiently.

(1) It can search around the given Pareto optimum individuals. (local search ability)
(2) It can search all over the feasible domain. (global search)
(3) It does not search needless local search (efficient search)

From these points of view, the simple island model is not good model for multi objective genetic algorithms. In the simple island model, all of the islands search the solutions in the same feasible domain. Therefore, the efficient search can not be performed. At the same time, the adequate local search can not be performed, because population size is smaller than that of the single population model.

These disadvantages are occurred, because each island search the same feasible domain. Therefore, in the introduced the DRGA, the searching domain is different in each island. The performance of the DRGA is as good as that of the one population model and the DRGA has the following advantages.

The flow of Distributed Range Genetic Algorithm is explained as follows.

- **Step 1** Initial population (population size is N) is produced randomly. All the design variables that are shown with the individuals satisfy the constraints.
- **Step 2** The individuals whose ranks are 1 are chosen.
- **Step 3** The individuals are sorted by the values of focused objective function f_i . This focused objective function f_i is chosen in turn, and turned with the loop. Then, the individuals of number N/m are chosen in accordance with the value of this focused objective function f_i . As the result, there exist m the sub populations.
- **Step 4** In each sub population, the multi objective optimization has been performed. The multi objective optimization that is used in this paper is explained in the next section.
The end of each generation, the terminal condition is examined and the process is terminated when the condition is satisfied.
- **Step 5** After the multi objective optimization has been performed for k generations, the process is backed to step 3. This generation k is called the sort interval.

In this study, the number of distribution m and the sort interval k is determined at first. In Fig. 2, the concept of the DRGA is shown. In Fig. 2, there two objective functions and the individuals are divided into three by the value of the focused objective function f_1.

The sub population of the DRGA is determined by the area with respect to the focused objective function. This mechanism is supposed to functions as the sharing. Therefore, the derived Pareto optimum solutions of the DRGA might have the high diversity.

Using the DRGA in parallel processings, the following items can be expected.

(1) Speed up of the operations

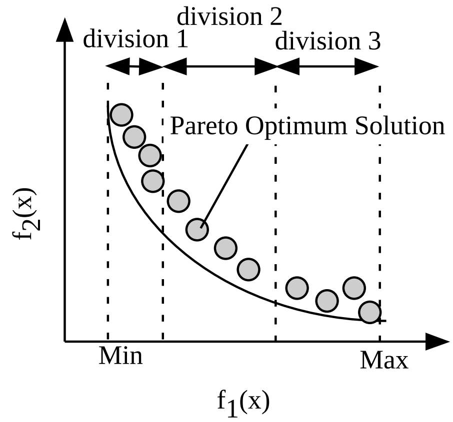

Fig. 2 DRGA

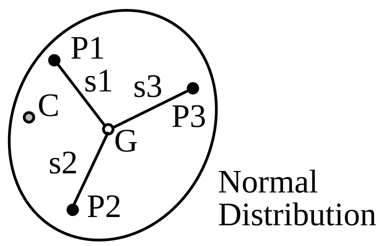

Fig. 3 Gravity crossover (three variables)

(2) Increase of the availability of memory. When there are many objective functions, the many points are necessary.

3.2 Configuration of Genetic Algorithm

3.2.1 Expression of individuals

In genetic algorithms, usually, the individuals are shown in bit line. However, the example problems are the real value problems. Therefore, the individuals are shown in the vectors of the real values in this study. For example, the individuals are shown like

$$a_1 = \{0.02, 10.03, \cdots, 7.52\}. \quad (4)$$

Each elements show the values of the design variables.

3.2.2 Crossover

Because the individuals are shown in real values, the crossover operation is performed in the following way. This is expanded method of Tutui's study (Tutui and Ghosh, 1998).

At first, when there are n design variables, the arbitrary $n + 1$individuals (parents) are chosen. Then the gravity G of these n individuals are determined.

With the n individuals and the gravity G, the new individuals (children) C is generated with the following equation,

$$\overrightarrow{C} = \overrightarrow{G} + \sum_{i=1}^{N}(N(0, s_i^2)\overrightarrow{GP_i}) \quad (5)$$

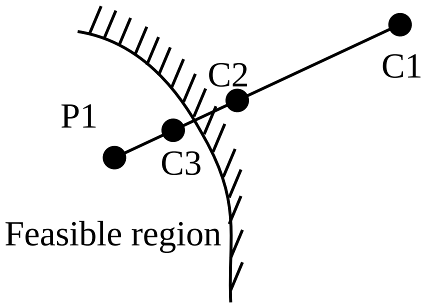

Fig. 4 Pull in method to the feasible domain

where $N(0, s_i^2)$ is the normal distributed function and the dispersion is equal to the distance between the individual i and the gravity G.

The crossover that uses the gravity and the multi parents are called the gravity crossover and the concept is shown in Fig. 3.

With these expression of the individuals and the crossover method, the searching in the real value space is performed quickly. More than that, when the parents exist widely, the generated children also exist widely. When the parents are close to each other, the children are also close th the parents. Therefore, the children have the heredity of the parents' characteristics.

When the new individuals does not satisfy the constraints, the substitutive individuals are derived with the following equation.

$$\overrightarrow{C_{new}^{(i)}} = \overrightarrow{P_1} + \alpha^i \overrightarrow{P_1 C_i} \tag{6}$$

Here, $\alpha^i = 0.5$. When the derived substitutive individual does not satisfy the constraints, i is increased and find the other substitutive individual. This operation is performed until the individual satisfies the constraints. The concept of this is shown in Fig. 4.

3.2.3 Other genetic operations

In this study, the mutation is not performed. The terminal condition that is explained in the next is effected by the mutation. The crossover method that is explained before can operate the same way of mutation.

The selection is a elite selection and the all the individuals whose ranking is 1 are selected. When the population size is over the certain number, the individuals are chosen by the roulette selection with the fitness values. The fitness values are determined by the sharing and the fitness value f_i^s is shown with the following equation,

$$f_i^s = \frac{1}{m+1} \tag{7}$$

where m is the number of individuals that are in the area of the sharing radius. Therefore, when $m = 0$, the fitness value f_i^s is equal to 1. When m becomes bigger, the fitness value f_i^s becomes smaller.

3.2.4 Terminal condition

Many researchers used the number of generation as the terminal conditions. However, this condition is not practical, because the optimum generation can be determined after the solutions are derived.

In this step, the terminal condition is determined with the following steps.

- **Step 1**
  The individuals a of generation G are saved. These individuals should be rank 1.
- **Step 2**
  Just after the elite selection where the individuals that are rank 1 is remained, the individuals b are save. Then the comparison between a and b is performed. When m% of a are in b, the terminal condition is satisfied.
- **Step 3**
  The condition of step 2 is satisfied with the continuous k times, the algorithms is terminated.

From these steps, the movement of the Pareto frontier is confirmed. This terminal condition is equal to the situation that the movement of the Pareto frontier is very small. In this procedure, m and k are the parameters that designer should be set and these parameters are not effected by the type of the problems. In this study, these parameters are set $m = 98$ and $k = 3$ respectively.

4. NUMERICAL EXAMPLES

4.1 Test Function

The proposed DRGA was adapted to the following some test functions. By making the DRGA adapted to these test functions, the validity of the DRGA and the characteristics of the solutions are discussed. From example 1 to example 3 are the problems that Tamaki et al. used in their research (Tamakei, et al., 1995). The real Pareto optimum solutions of these problems are given. The example 4 is the problem that Veldhuizen (Veldhuizen and Lamont, 1999) used in his research and this example is a very difficult test function to derive the real Pareto optimum solutions. The real Pareto solutions of this test function is not given.

Example 1

$$f_1(x) = x_1^2 - x_2 \tag{8a}$$

$$f_2(x) = -\frac{1}{2}x_i - x_2 - 1 \tag{8b}$$

$$g_1(x) = \frac{1}{2}x_1 + x_2 - \frac{13}{2} \leq 0 \tag{8c}$$

$$g_2(x) = \frac{1}{2}x_1 + x_2 - \frac{13}{2} \leq 0 \tag{8d}$$

$$g_3(x) = \frac{1}{2}x_1 + x_2 - \frac{13}{2} \leq 0 \tag{8e}$$

$$g_4(x) = x_1 \geq 0 \tag{8f}$$

$$g_5(x) = x_2 \geq 0 \tag{8g}$$

Example 2

$$f_1(x) = -2x_1 + x_2 \tag{9a}$$

$$f_2(x) = x_2 \tag{9b}$$

$$g_1(x) = x_1^2 - x_2 \leq 0 \tag{9c}$$

$$g_2(x) = x_1 \geq 0 \tag{9d}$$

$$g_3(x) = x_2 - 1 \leq 0 \tag{9e}$$

(9f)

Example 3

$$f_1(x) = 2\sqrt{x_1} \tag{10a}$$

$$f_2(x) = x_1(1 - x_2) + 5 \tag{10b}$$

$$g_1(x) = x_1 - 1 \geq 0 \tag{10c}$$

$$g_2(x) = 4 - x_1 \geq 0 \tag{10d}$$

$$g_3(x) = x_2 - 1 \geq 0 \tag{10e}$$

$$g_4(x) = 2 - x_2 \geq 0 \tag{10f}$$

Example 4

$$f_1(x) = 0.5(x_1^2 + x_2^2) + \sin(x_1^2 + x_2^2) \tag{11a}$$

$$f_2(x) = \frac{(3x_1 - 2x_2 + 4)^2}{8} + \frac{(x_1 - x_2 + 1)^2}{27} + 15 \tag{11b}$$

$$f_3(x) = \frac{1}{x_1^2 + x_2^2 + 1} - 1.1\exp(-x_1^2 - x_2^2) \tag{11c}$$

$$g_1(x) = x_1 \geq -3 \tag{11d}$$

$$g_2(x) = x_2 \leq 3 \tag{11e}$$

4.2 Parameter Settings

There exist many parameters in multi objective GA. In this study, the solutions of the DRGA are compared with those of single population GA (SGA) and distributed GA (DGA). In table 1 , the used parameters of crossover rate, mutation rate, migration interval and migration rate are summarized.

The parameters of population size and the sharing radius effects the accuracy of solutions. It is very difficult to find the optimum population size and the sharing radius (Coello, 1999). Therefore, we used the 5 cases that are summarized in table 2 . The sharing range is the sub parameter that determined the sharing radius. The sharing radius is derived with the sharing range as follows. Find the two individuals whose distance is the longest in the Pareto optimum individuals. Then, the sharing range is derived by dividing the distance by the sharing range.

Table 1 Parameters

	SGA	DGA	DRGA
Crossover rate		1.0	
Mutation rate		0.0	
Number of island		5	
Migration interval (Sort interval)	—	5	
Migration rate	—	0.1	—

Table 2 Population size and sharing range

case	population size	sharing range
Case 1	50	25
Case 2	100	50
Case 3	200	100
Case 7	500	250
Case 5	1000	500

4.3 The Results of Numerical Examples

The results of 4 types of the numerical examples that are explained in the former sub section are shown in this section. The results of the DRGA is compared the results of the simple population model (SGA) and the distributed population model (DGA). All the results are the average of 10 random trials.

4.3.1 Example 1

Example 1 is the problem that has two objective functions and it is rather easy to derive the Pareto optimum solutions.

The errors of the derived Pareto optimum individuals are shown in Fig. 5.

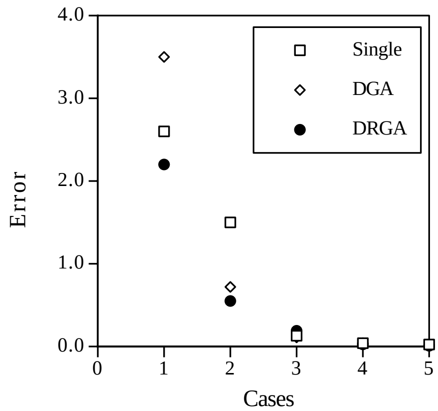

Fig. 5 Example 1, error

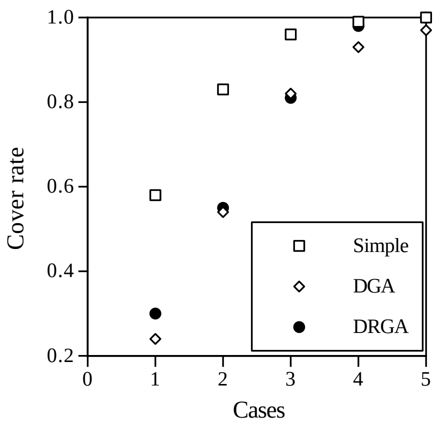

Fig. 6 Cover rate (Example 1)

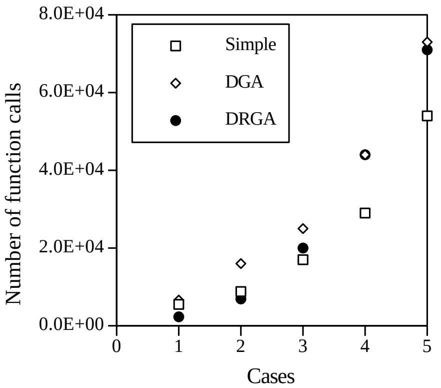

Fig. 7 Number of function calls (Example 1)

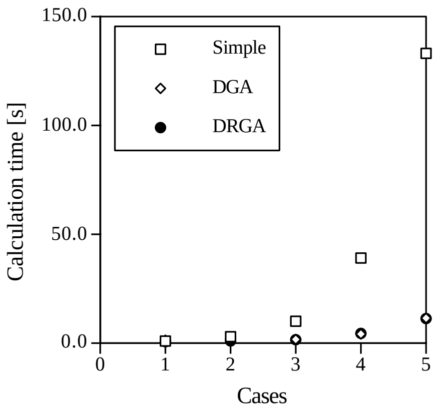

Fig. 8 Total calculation time (Example 1)

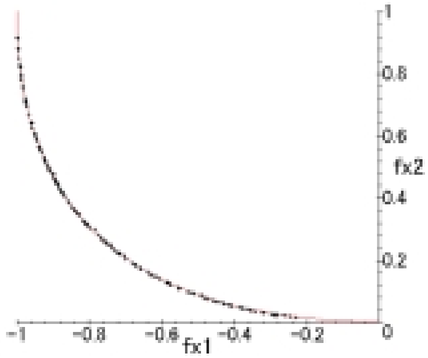

Fig. 9 Pareto optimum solutions (Example 2, SGA, Case 5)

From Fig. 5, it is found that the solutions of DRGA have high accuracy compared to the other models when the population size is small. This result means that the DRGA is effective model to find the Pareto solutions not only in the parallel processings but also in the sequential processings.

When the population size is big, every models can derive the solutions that have high accuracy. This comes from the termination condition that is explained the former section. These tendencies are the same in the other numerical examples.

In Fig. 6, the cover rates are shown. The solutions of SGA have good results. There is no great differences with the solutions of the DRGA and those of the DGA. This is because that this problem is easy to derive the Pareto solutions. The SGA can find the better Pareto optimum individuals with any parameters. There is not big differences between the results of the DRGA and the DGA because this problem is rather easy to find the Pareto optimum solution.

In this study, the problems are solved with 5 processors. Though, the coding is not enough developed, the speed up can be obtained when the population size is big. For example, comparing the DRGA and the SGA of case 5, the calculation time of the SGA is 5 times bigger than that of the DRGA. In the DRGA, the population is divided into sub populations and the number of individuals that concerned with the sharing or the calculating of ranking decrease. These distribution effects are the almost same in the other examples. Therefore, DRGA can speed up by parallel when the population size is big.

4.3.2 Example 2

Example 2 is the problem whose Pareto solutions are rather difficult to derive. The Pareto optimum individuals of SGA, DGA and DRGA are shown in Fig. 9, 10 11 respectively.

In Fig. 12, the cover rates are shown.

With the figures of the distribution of the Pareto optimum individuals 9 , 10, 11 and Fig. 12, it is found that this is the problem which is difficult to find the Pareto optimum solutions. Even SGA can not derived the solutions whose cover rate is close to 1.0. The cover rate of the DGA is very bad. The cover rate of the DRGA is not good but not bad as that of the DGA. Therefore, it can be said that the DRGA is better parallel model of multi objective GA in parallel compared to the DGA.

The number of objective function calls are shown in Fig. 13.

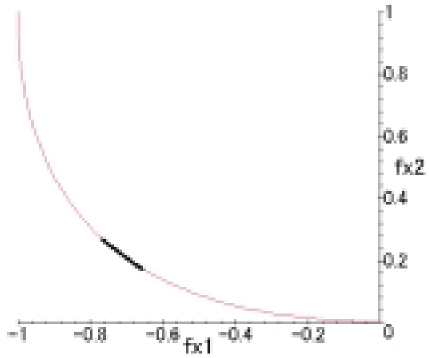

Fig. 10 Pareto optimum solutions(Example 2, DGA, Case 5)

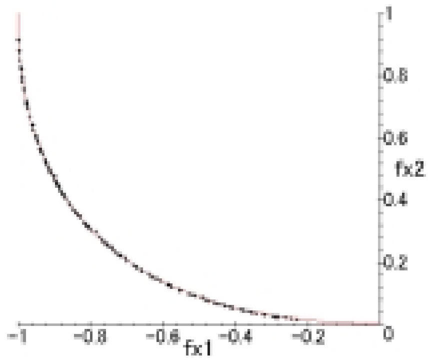

Fig. 11 Pareto optimum solutions(Example 2, DRGA, Case 5)

In this example, the DGA needs a lot of function call. On the other hand , the DRGA can derive the solutions with almost the same number of the function calls of the SGA.

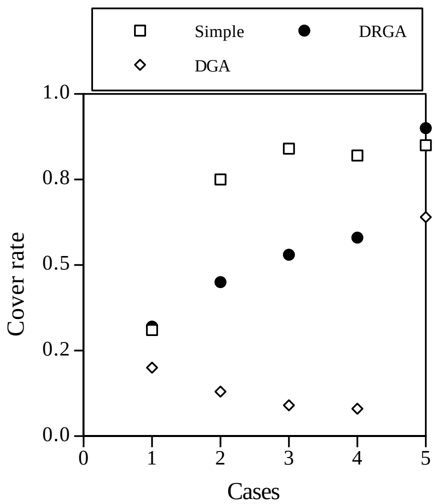

Fig. 12 Cover rate (Example 2)

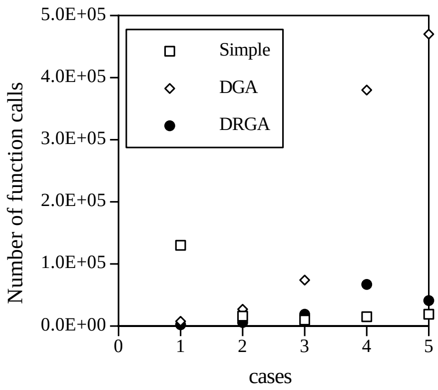

Fig. 13 Number of objective function calls (Example 2)

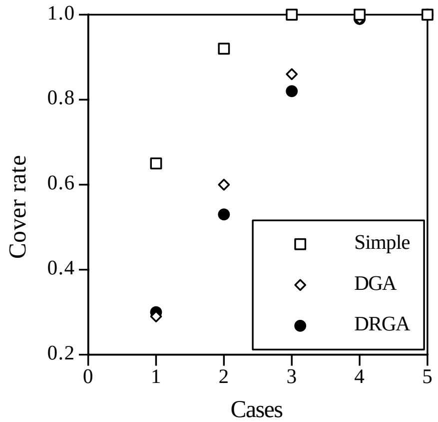

Fig. 14 Cover rate (Example 3)

4.3.3 Example 3

This example is the problem whose shape of the Pareto solutions are the concave. The cover rate of the derived Pareto optimum individuals are shown in Fig. 14.

Although, the shape of the Pareto solutions are the concave, this is the problem whose Pareto solutions are rather easy to derive. Therefore, any model can find the good Pareto solutions when there are enough number of individuals. When the population size is small, the results of SGA is good. This results are the same as those of example 1. It can be said that the shape of the Pareto solutions do not effect to the errors or the cover rate of the Pareto optimum individuals.

4.3.4 Example 4

There are three objective functions in example 4 and it is very difficult to find the real Pareto solutions. The Pareto optimum individuals of case 5 that are derived by SGA, DGA and DRGA are shown in 15, 16, 17, respectively.

The error of this test function can not be derived, because the real Pareto solution can not be found.

The cover rate of the solutions are shown in 18.

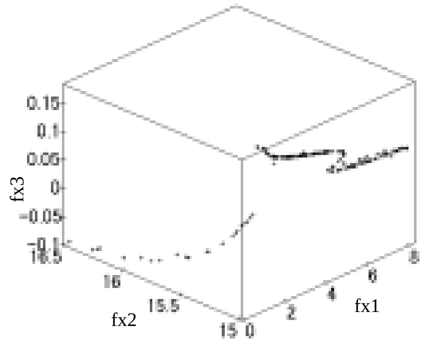

Fig. 15 Pareto optimum individuals (Example 4, SGA, Case 5)

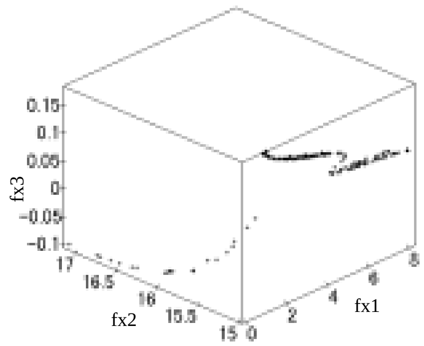

Fig. 16 Pareto optimum individuals (Example 4, DGA, Case 5)

From Fig. 18, it is obvious that the Pareto optimum individuals of the DRGA is better than those of the SGA. This is because that this problem is very difficult to find the Pareto optimum solutions. Therefore, it needs an adequate local search all over the feasible domain. It can be said, the DRGA is suitable for this problem.

In Fig. 19, the number of objective function calls are shown. There is no remarkable characteristics except the SGA has the smallest values. This tendency is also come from the fact that this problem is difficult to derive the Pareto optimum solutions.

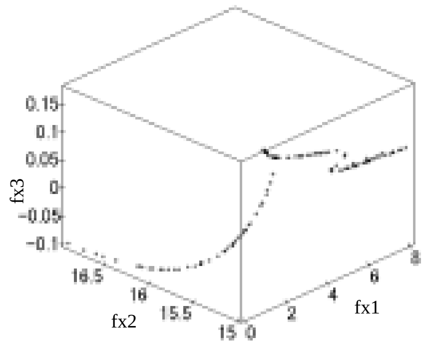

Fig. 17 Pareto optimum individuals (Example 4, DRGA, Case 5)

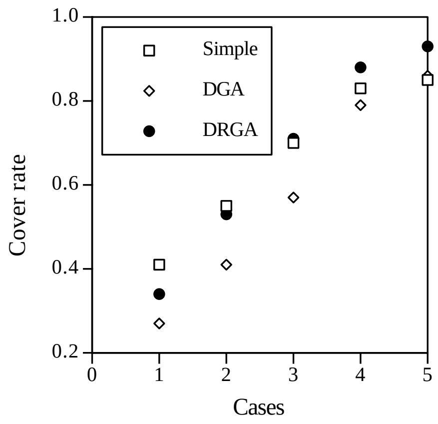

Fig. 18 Cover rate (Example 4)

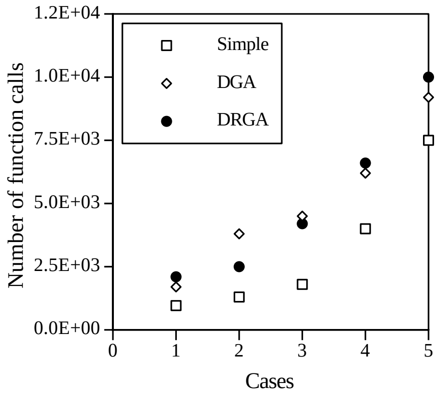

Fig. 19 Number of objective function calls (Example 4)

5. Conclusions

In this study, the new parallel model of genetic algorithms in multi objective problems. This new model is called Divided Range Genetic Algorithm: DRGA. In this model, population is divided into sub populations with along to the value of the focused objective function. The effectiveness and validity are discussed through the typical numerical test functions.

Through the typical numerical test functions, it became clear that the DRGA has the following characteristics.

(1) In some cases, the solutions that derived by the DRGA is better than those of the single population model.
(2) When it is compared with the island model, the DRGA model is especially effective in the problems where it is difficult to find the Pareto optimum solutions.
(3) In the operation of sharing, the distances between the every two individuals are derived. Therefore,

it takes much time, when many individuals exist. The DRGA is one of the distributed population models and the population is divided into sub populations. This distributed processing leads to speed up in the DRGA.

(4) In DRGA, every sort has been performed by the value of the focused objective function. This focused objective function is chosen in turn, and turned with the loop. It can be thought that this operation plays the part of the sharing.

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